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I Semester M.Sc. Examination, February - 2020
(CBCS Y2K17)(2017 - 18 and Onwards Scheme)

MATHEMATICS

M-107SC : Mathematical Analysis

Time : 3 Hours　　　　Max. Marks : 70

Instructions : *(i) Answer **any five** questions.*
*(ii) **All** questions carry **equal** marks.*

1. (a) Show that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(C)$ is closed in X for every closed set C in Y.

(b) Let f and g be continuous functions on a metric space X. Then prove that $f.g$ and f/g $(g \neq 0)$ are continuous on X. **7+7**

2. (a) Suppose f is continuous mapping of a compact metric space X into metric space Y. Then show that image of X under f is compact in Y.

(b) Let E be a non-compact set in **R**. Then prove that :

(i) there exists a continuous function on E which is not bounded,

(ii) there exists a continuous and bounded function on E which has no maximum. **7+7**

3. (a) State and prove generalized mean value theorem.

(b) Suppose f is a real differentiable function on [a, b] and suppose $f'(a) < \lambda < f'(b)$. Then show that there exists a point $x \in (a, b)$ such that $f'(x) = \lambda$.

(c) Discuss the continuity of the following function f at '0' defined by :

$$f(x)=\begin{cases} \sin 1/x & ; \ x \neq 0 \\ 0 & ; \ x=0 \end{cases}$$

(Assume that $\sin x$ is continuous function on **R**). **6+5+3**

P.T.O.

4. (a) If f is continuous on [a, b], $f'(x)$ exists at some point $x \in [a, b]$, g is defined on an interval I which contains the range of f and g is differentiable at a point $f(x)$ and if $h(t) = g(f(t))$, $(a \le t \le b)$, then show that h is differentiable at x and further prove that $h'(x) = g'(f(x)) \cdot f'(x)$.

(b) Suppose f is continuous real function on a compact metric space X. Then show that f is bounded and attains its bounds. **7+7**

5. (a) Suppose the sequence $\{x_n\}$ is monotonic. Then show that the sequence $\{x_n\}$ converges if and only if it is bounded.

(b) Prove that a sequence of real numbers converges if and only if it is a Cauchy sequence.

(c) If P > 0 then show that $\lim_{n \to \infty} \frac{1}{n^P} = 0$. **4+6+4**

6. (a) If $a_n > 0$, $b_n > 0$ and $\frac{a_{n+1}}{a_n} \le \frac{b_{n+1}}{b_n}$ for n = 1, 2, 3, ..., and if $\sum_{n=1}^{\infty} b_n$ converges then show that $\sum_{n=1}^{\infty} a_n$ also convergent.

(b) Show that $\sum_{n=1}^{\infty} \frac{1}{n^P}$ converges if P > 1 and diverges $P \le 1$.

(c) Suppose $a_n > 0$, $b_n > 0$, $\forall n \ge 1$ $(n \in \mathbf{N})$ and $\lim_{n \to \infty} \frac{a_n}{b_n} = l$, where $l \ne 0$ and finite. Then show that $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ behave alike. **4+5+5**

7. (a) For any sequence $\{C_n\}$ of positive real numbers, show that :

(i) $\overline{\lim_{n \to \infty}} \sqrt[n]{C_n} \le \overline{\lim_{n \to \infty}} \frac{C_{n+1}}{C_n}$

(ii) $\underline{\lim_{n \to \infty}} \sqrt[n]{C_n} \ge \underline{\lim_{n \to \infty}} \frac{C_{n+1}}{C_n}$

(b) State and prove logarithmic test for series.

(c) State and prove Abel's test. **6+4+4**

8. (a) State and prove Merten's theorem.

(b) Suppose $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are two absolutely convergent series and converges to A and B respectively then prove that the series $\sum_{n=0}^{\infty} c_n = C$ where $c_n = \sum_{K=0}^{n} a_n b_{n-k}$, (n=0, 1, 2, ...) is convergent and converges to its sum $C = A \cdot B$.

(c) Suppose $\sum_{n=0}^{\infty} a_n$ is the divergent series of positive terms. Then prove that $\sum_{n=1}^{\infty} a_n/s_n$ diverges where $s_n = a_1 + a_2 + \ldots + a_n$. **6+4+4**

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